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Seminar 37 — Reduced Order Modeling for HVAC&R Systems and their Components

Reduced-Order Modeling and Fast Simulation of Transient Vapor Compression Cycles

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Learning Objectives

- Understand technical challenges associated with implementing nonlinear model order reduction approaches to Vapor Compression Cycle (VCC) applications.
- Become familiar with stabilization schemes associated with nonlinear reduced-order models.
- Understand a nonlinear model order reduction (MOR) framework for generating reduced-order VCC models that can achieve fast transient simulation.

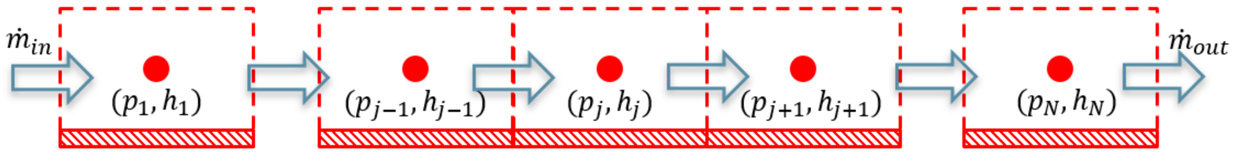
Outlines

- Introduction
  - Motivation
  - Model Order Reduction
- Methodologies
  - Reformulation of Finite Volume Model
  - Construction of Nonlinear Reduced-Order VCC Model
  - Stabilization of Reduced-Order Model
- Model Validations
- Conclusions

Motivation

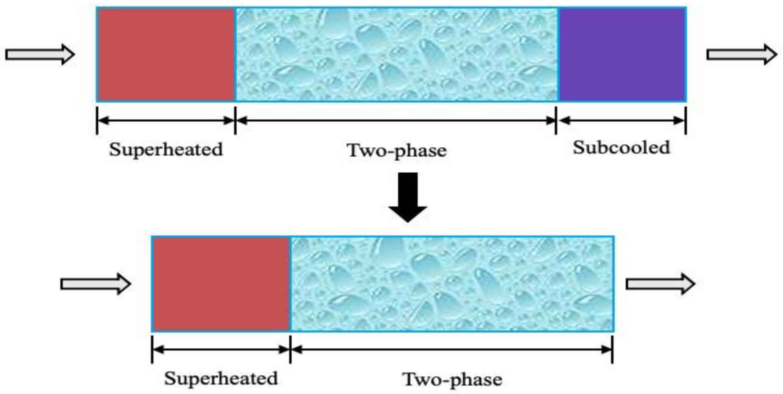
Dynamic modeling of vapor compression cycles (VCC)

Finite Volume (FV)



- ✓ High-fidelity
- ✓ Simpler to derive and implement
- ✓ More robust for fast transients (e.g., start-up)
- ➢ High-order ODE system

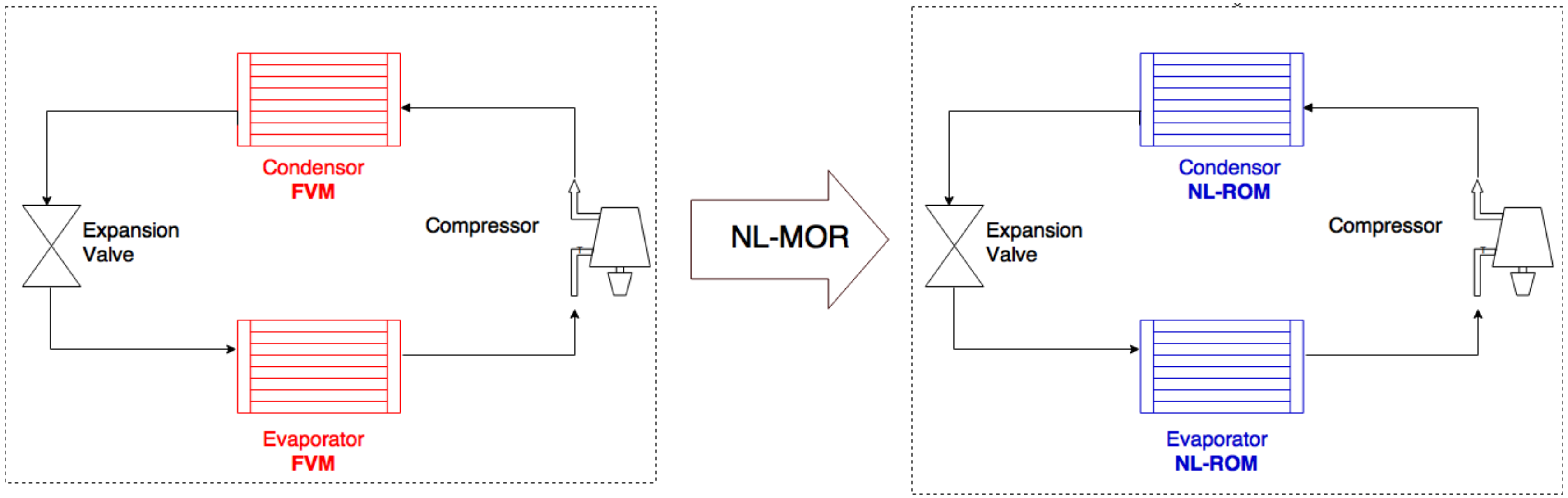
Moving Boundary (MB)



- ✓ Low-dimensional
- ✓ Faster execution
- ➢ Numerical issue in switching model representations[1]

[1] Kim, D., Ma, J., Braun, J. E., & Groll, E. A. (2021). Fuzzy modeling approach for transient vapor compression and expansion cycle simulation. *International Journal of Refrigeration*, *121*, 114-125.

Model Order Reduction



Conventional Dynamic Simulation Scheme

Proposed Fast Simulation Scheme

Full-order (high-dimensional) FV heat exchanger (HX) model $\dot{x} = f(x,u), y = g(x,u), \quad x \in R^N$

MOR

Reduced-order HX model $\dot{x}_r = f_r(x_r,u), y = g_r(x_r,u), \quad x_r \in R^{N_r} \quad N_r \ll N$

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Reformulation of FV Model

Standard FV formulation: (p, h) as states

Mass balances: $$\left(\frac{\partial \rho_j}{\partial p}\right)\frac{dp}{dt} + \left(\frac{\partial \rho_j}{\partial h}\right)\frac{dh_j}{dt} = \frac{1}{V_j}(\dot{m}_{j-1} - \dot{m}_j)$$

Energy balances: $$\left(h_j\frac{\partial \rho_j}{\partial p} - 1\right)\frac{dp}{dt} + \left(h_j\frac{\partial \rho_j}{\partial h} + \rho_j\right)\frac{dh_j}{dt} = \frac{1}{V_j}(\dot{m}_{j-1}h_{j-1} - \dot{m}_j h_j - \dot{Q}_j)$$

$$\boldsymbol{x} = [p \quad h_1 \quad h_2 \quad \dots \quad h_N]^T$$
$$\boldsymbol{u} = [\dot{m}_{in} \quad \dot{m}_{out} \quad h_{in}]^T$$

⟺

$$\boldsymbol{E}(\boldsymbol{x})\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

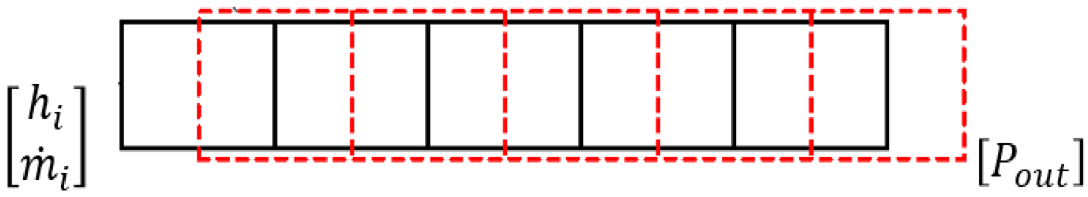
Goal: reformulate the standard FV model into a standard ODE structure.

$$\boldsymbol{E}(\boldsymbol{x})\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \quad \Longrightarrow \quad \dot{\boldsymbol{x}} = \tilde{\boldsymbol{f}}(\boldsymbol{x}, \boldsymbol{u})$$

Reformulation of FV Model

To have a FV-HX model in standard ODE form

- Choose density ρ, internal energy u and interface mass flow rates $\dot{m}$ as dynamic states.
- Integrate the momentum balances into governing equations.
- Apply a staggered grid for discretization.



Mass balances:

$$\frac{d\rho_j}{dt} = \frac{1}{V_j}(\dot{m}_k - \dot{m}_{k+1})$$

Energy balances:

$$\frac{du_j}{dt} = \frac{1}{V_j \rho_j}(\dot{m}_k h_{j-1} - \dot{m}_{k+1} h_j - \dot{Q}_{r,j} + u_j(\dot{m}_{k+1} - \dot{m}_k))$$

Momentum balances:

$$\frac{d\dot{m}_k}{dt} = \frac{1}{L}\left(\dot{m}_{j-1} v_{j-1} - \dot{m}_j v_j + \frac{A_c}{L}(P_{j-1} - P_j) - F_f\right)$$

$$\boldsymbol{x} = [\boldsymbol{\rho} \quad \boldsymbol{u} \quad \dot{\boldsymbol{m}}]^T$$

$$\boldsymbol{u} = [\dot{m}_{in} \quad h_{in} \quad p_{out}]^T$$

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$$

Full-order model (FOM)

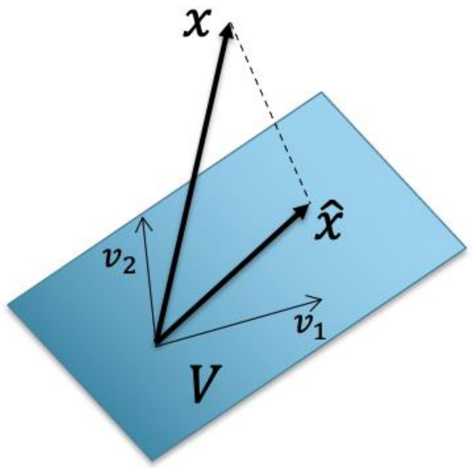
Construction of Reduced-Order Model

Approach: Proper Orthogonal Decomposition (POD)

1. Given a set of state trajectories (snapshots) of a dynamical system

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(t)} \\ | & | & & | \end{bmatrix}$$

2. Projection of states ***x*** onto an orthonormal basis ***V***



To obtain the reduced basis ***V***:

$$\min \sum_{j=1}^{t} \left\| x_j - VV^T x_j \right\|^2$$

$$\text{s.t.} \quad V^T V = I_k$$

$\hat{x} = VV^T x \qquad V \in R^{N \times k}$

is the orthogonal projection of ***x*** onto span of ***V***

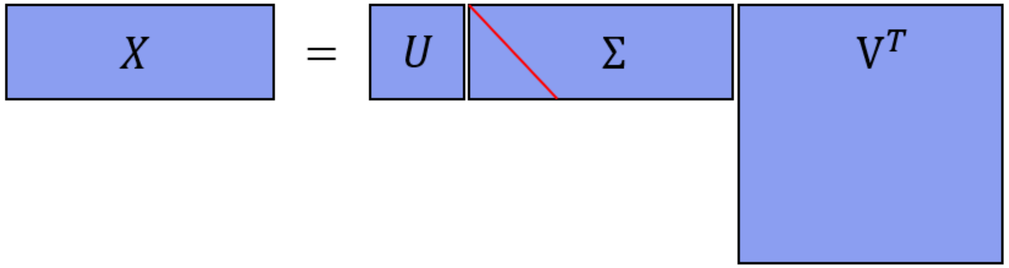
Proper Orthogonal Decomposition (POD)

Steps of generating POD reduced−order model

1. Snapshots of state trajectories:

$$X = \begin{bmatrix} | & | & & | \\ x^{(1)} & x^{(2)} & \dots & x^{(t)} \\ | & | & & | \end{bmatrix}$$

SVD



(Singular Value Decomposition (SVD) solves POD problems)

2. Truncation based on *k* dominant singular values:

$$U = \begin{bmatrix} | & | & & | & & | \\ u_1 & u_2 & \dots & u_k & \dots & u_N \\ | & | & & | & & | \end{bmatrix} \in R^{N \times N}$$

$$V = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \dots & u_k \\ | & | & & | \end{bmatrix} \in R^{N \times k}$$

Proper Orthogonal Decomposition (POD)

3. Represent full order state x by reduced coordinates a:

$$x \approx \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \dots & v_k \\ | & | & & | \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix} = Va$$

4. Projection of governing equations:

$$V\,\dot{a} = f(Va, u) \xrightarrow{\text{Galerkin Projection}} \dot{a} = V^T f(Va, u), a \in R^k$$

$k \ll N$

Computational Issue of Nonlinear MOR

A general nonlinear dynamical system: $\dot{x} = Ax + f(x,u) \quad x \in R^N$

POD ↓

$$\underbrace{\dot{a}}_{R^k} = \underbrace{V^T A V a}_{R^k} + V^T \underbrace{f(Va,u)}_{R^N} \quad a \in R^k \qquad k \ll N$$

Solution: Approximating nonlinear functions by DEIM[2]

A reduced-order basis ***S*** for nonlinear dynamics

Interpolation of a set of equations $P^T f(x)$

$$\hat{f}(x) = \underbrace{S(P^T S)^{-1}}_{\text{Pre-computed}} \underbrace{P^T f(x)}$$

$g(x) = [\ \ ]$ (rows selected from $f(x)$)

$g(x) \in R^m \quad m < N$

[2] Chaturantabut, S., & Sorensen, D. C. (2010). Nonlinear model reduction via discrete empirical interpolation. *SIAM Journal on Scientific Computing*, *32*(5), 2737-2764.

Stabilization of POD Reduced-Order Model

Consider an asymptotically stable LTI system $\dot{x} = Ax$

POD reduced order model

$$x \approx Va$$

$$\dot{a} = \tilde{A}a \quad \tilde{A} = V^T A V$$

Stability of the reduced model is basis and system variant

A stabilization scheme that operates on reduced bases[3]:

$\dot{x} = Ax$

$x \approx V_k a$ → $V_k \dot{a} = A V_k a$ → $W_k^T V_k \dot{a} = W_k^T A V_k a$

$V_k \in R^{N \times k}$ POD Search for a new basis

[3] Amsallem, D., & Farhat, C. (2012). Stabilization of projection-based reduced-order models. *International Journal for Numerical Methods in Engineering*, *91*(4), 358-377.

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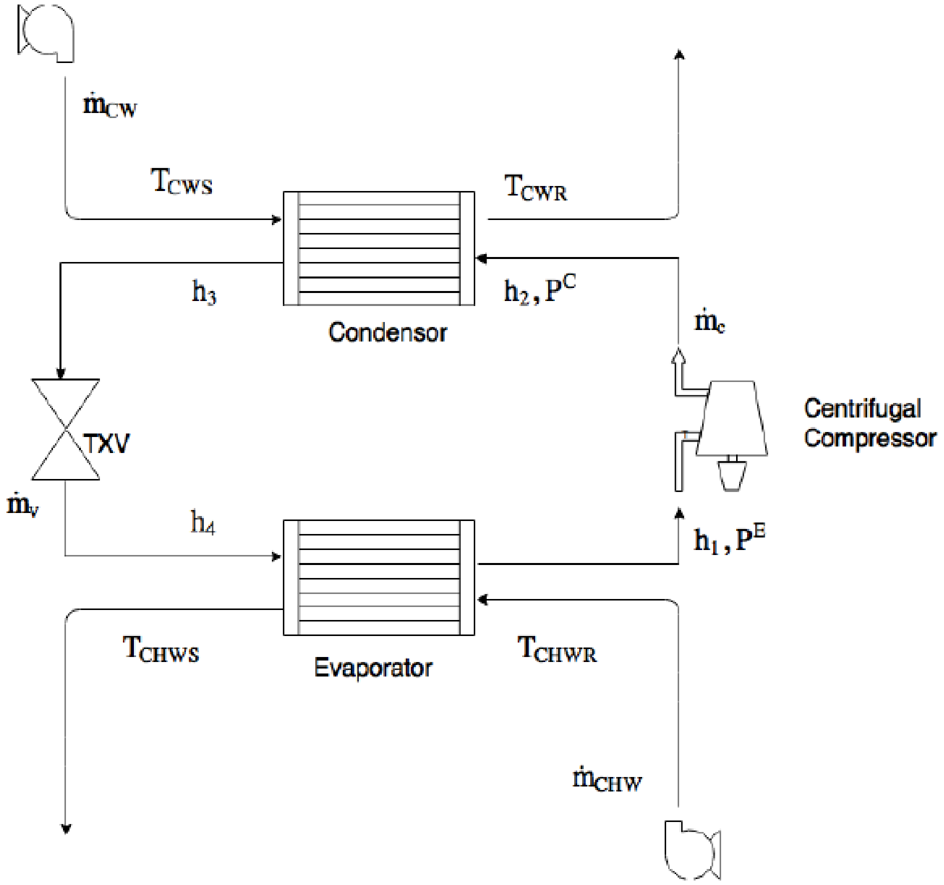
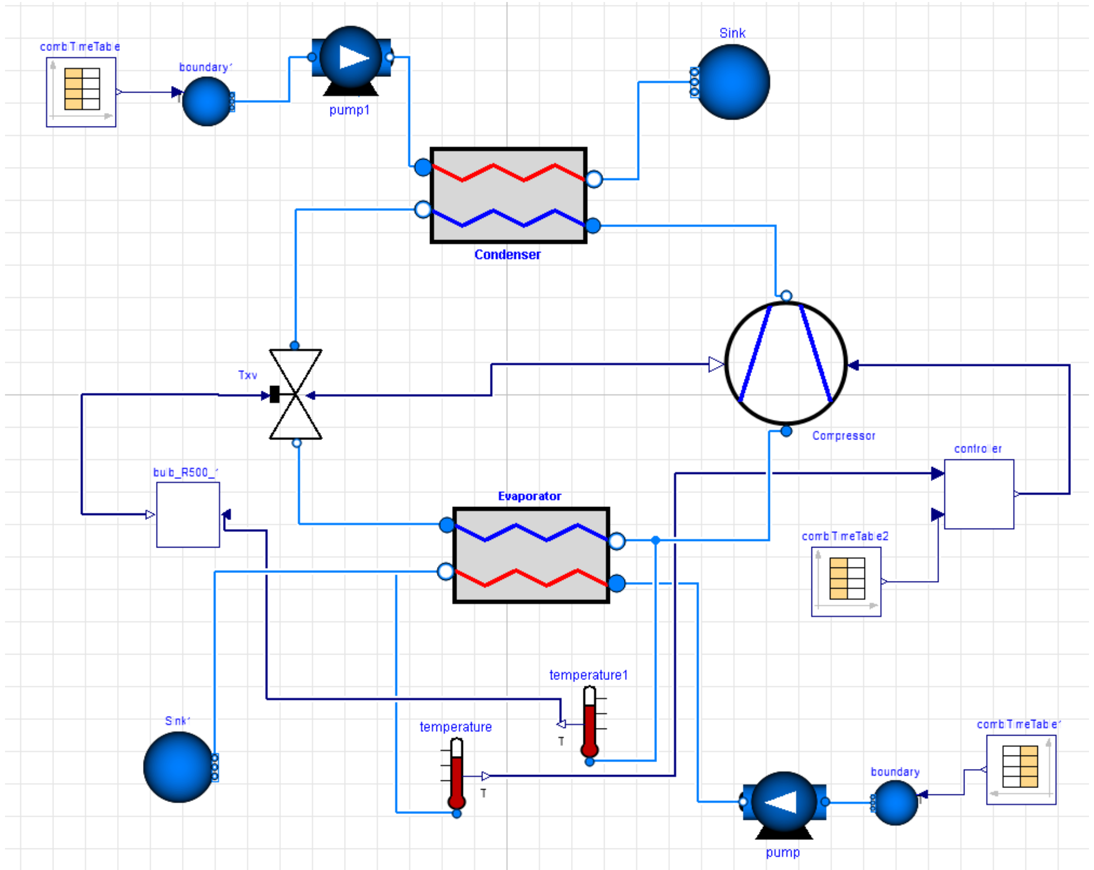
Case study: A 90-ton Centrifugal Chiller



Schematic of chiller system



Modelica model

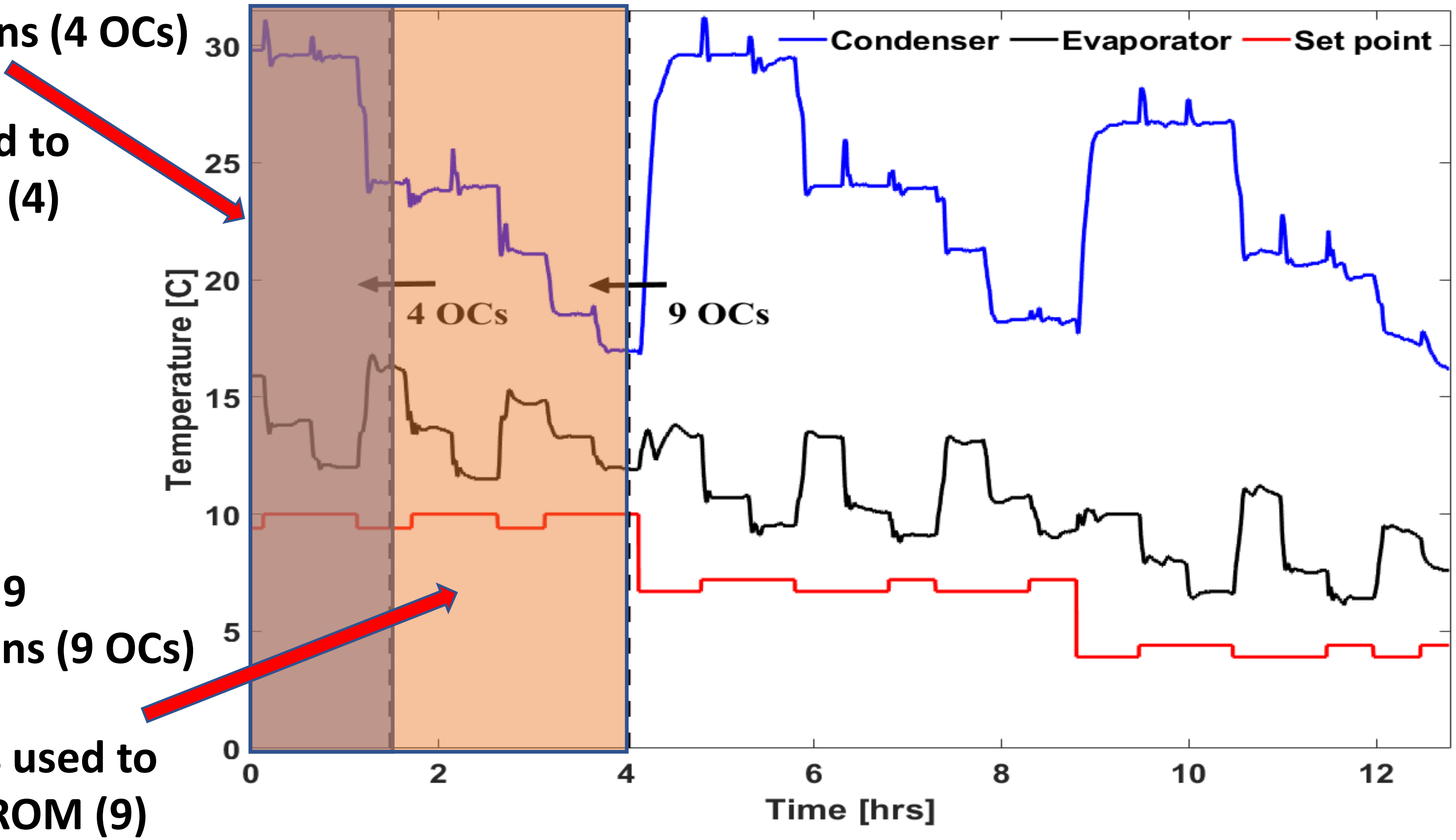
Case study: A 90-ton Centrifugal Chiller

Transients around 4 operating conditions (4 OCs)

Snapshots used to generate ROM (4)

Transients around 9 operating conditions (9 OCs)

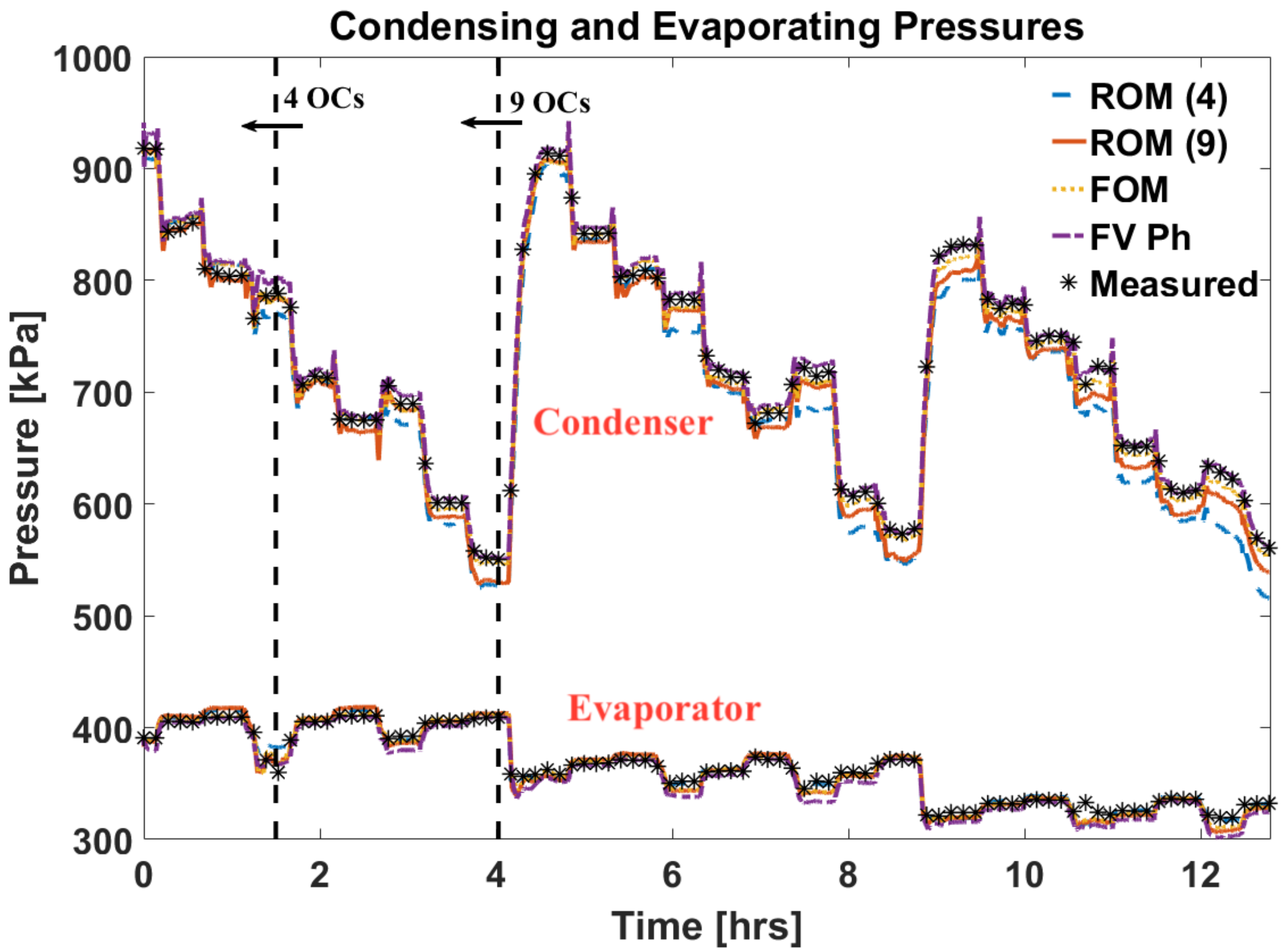
Snapshots used to generate ROM (9)



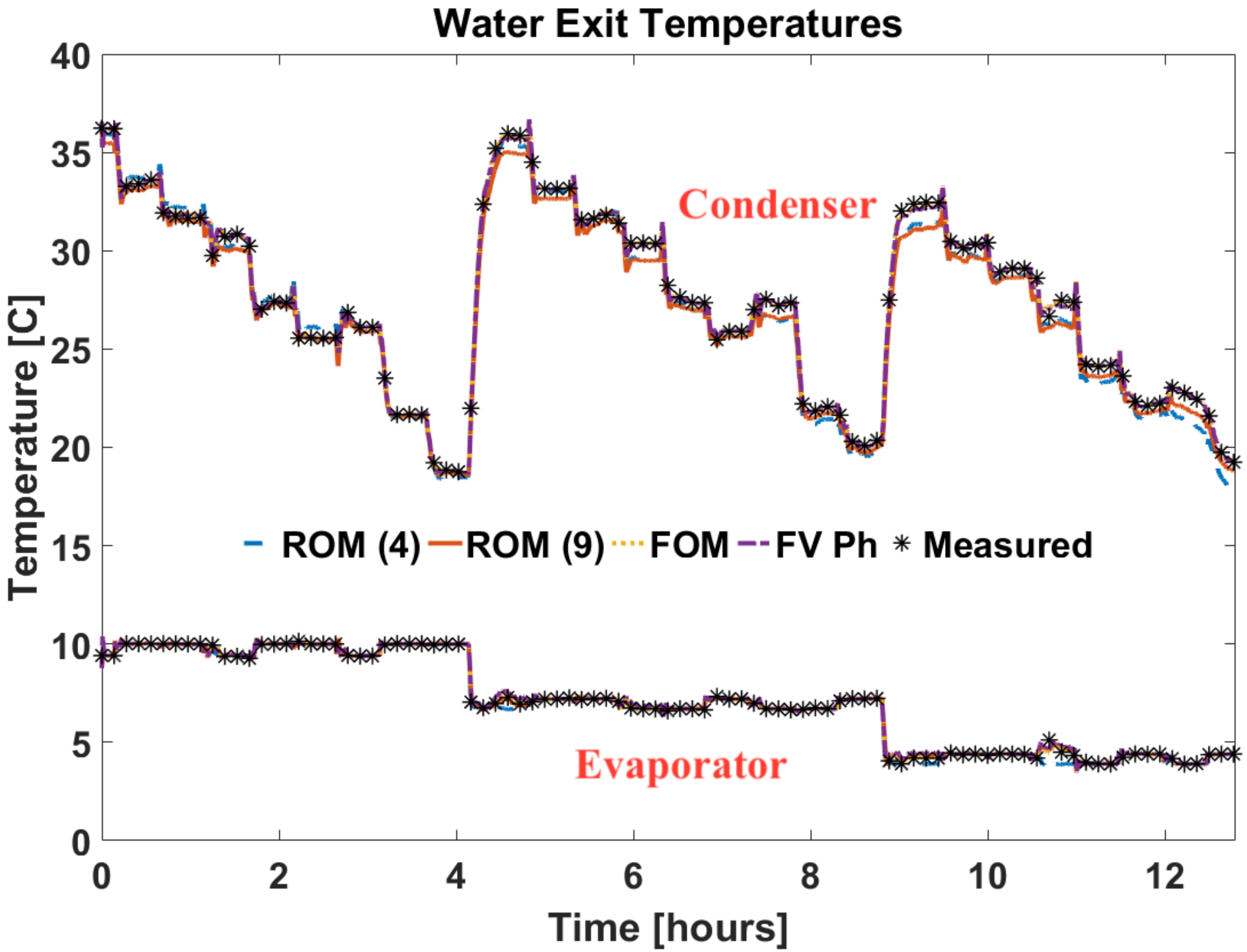
Control inputs used to drive load-change transients

Simulation Results: Refrigerant Pressures

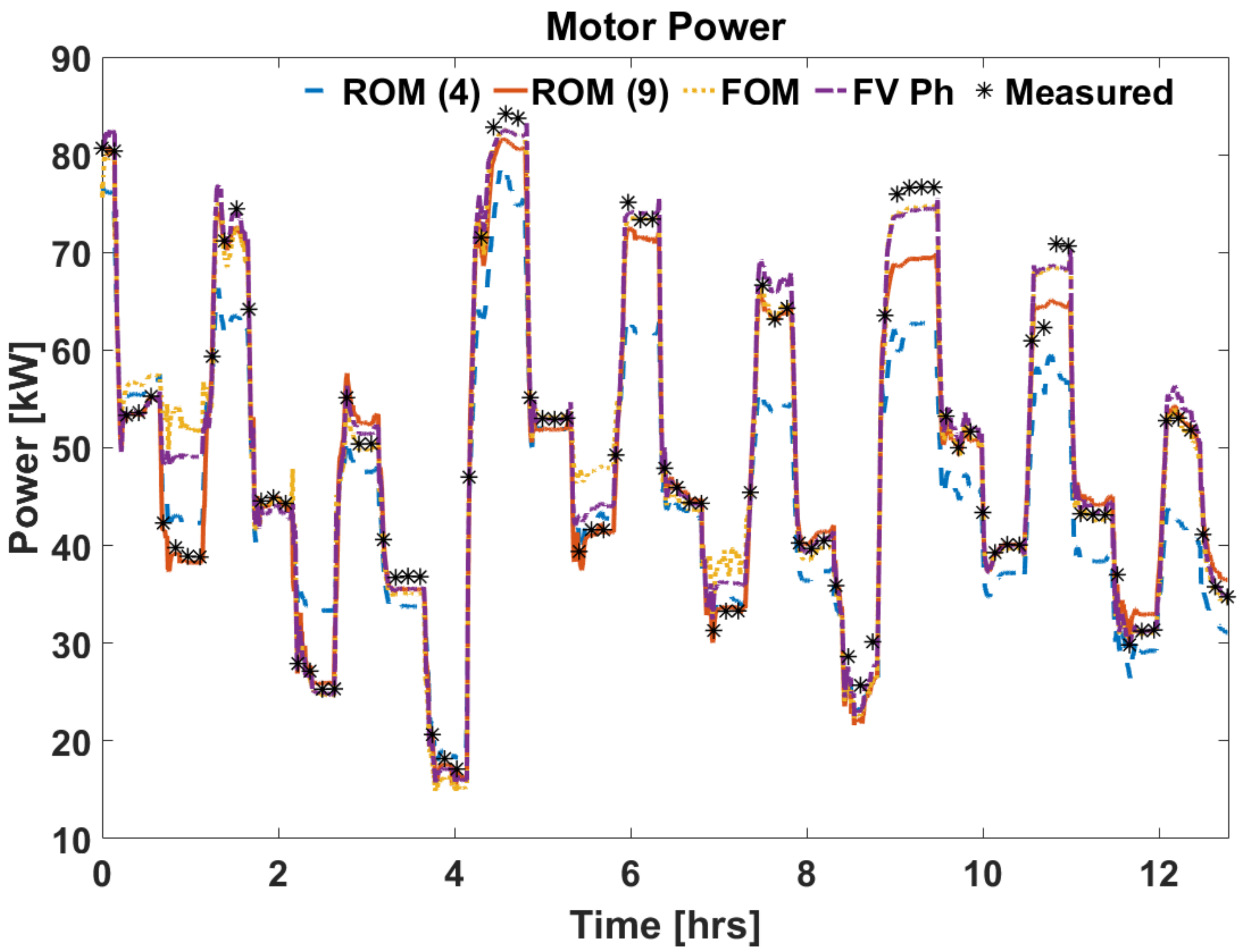
Comparisons of the reduced-order models ROM(4), ROM(9) with reformulated full-order model (FOM) and standard FV model (FV-Ph)



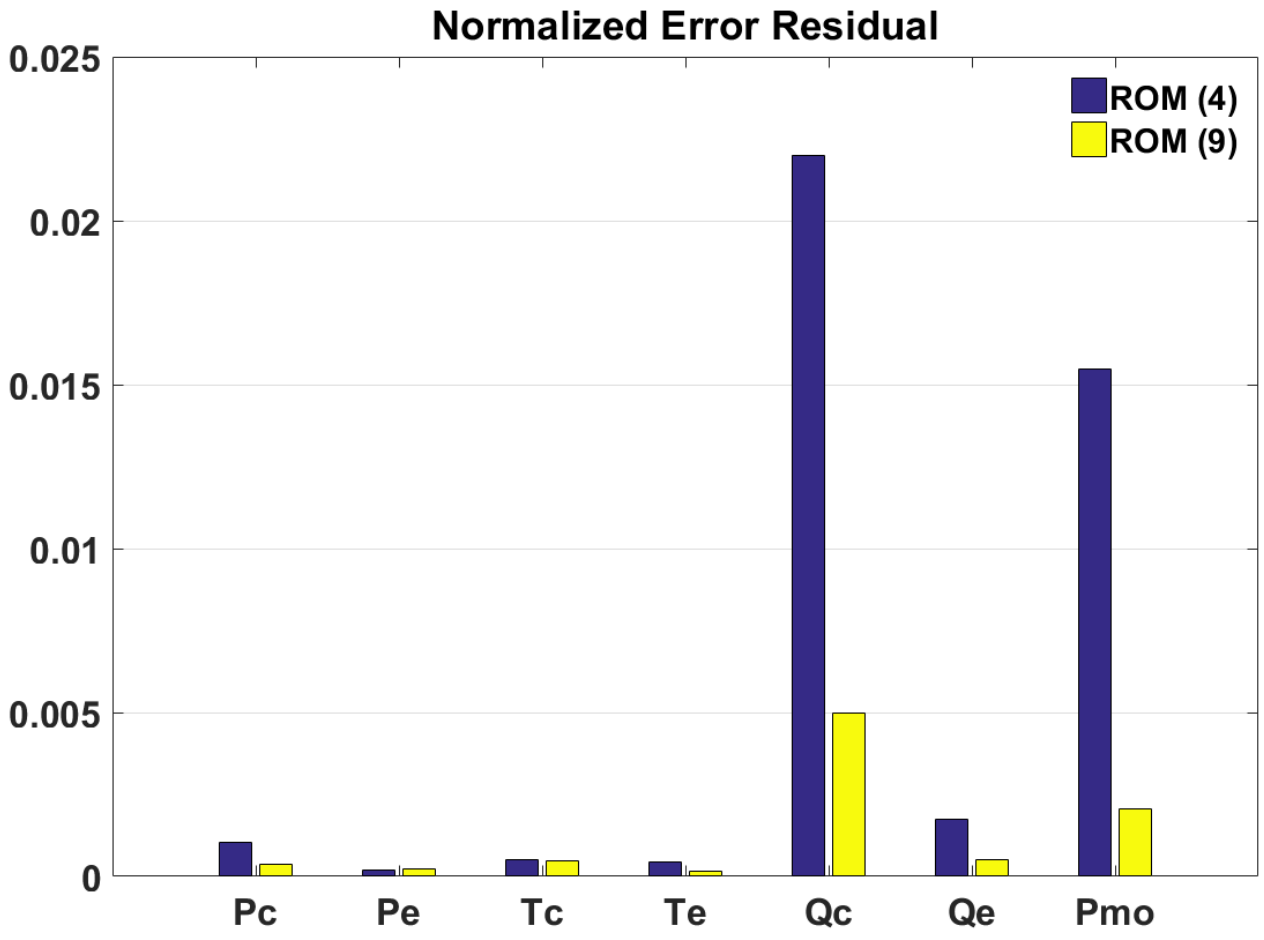
Simulation Results: Water Temperatures



Simulation Results: Motor Power



Performance of Reduced-Order Models



$$\mathrm{NER} = \frac{\sum_{k=1}^{t} \left(y_{\mathrm{predicted}}(k) - y_{\mathrm{measured}}(k)\right)^2}{\sum_{k=1}^{t} \left(y_{\mathrm{measured}}(k)\right)^2}$$

Prediction error of the refrigerant pressures (P_c, P_e), water exit temperatures (T_c, T_e), heat exchanger loads (Q_c, Q_e) and compressor motor power (P_{mo}).

Simulation Speed

Real time factor (RTF)

$$RTF = \frac{time\ taken\ to\ run\ the\ simulation}{time\ that\ is\ simulated}$$

Simulation	No. States	No. ODEs	RTF
Standard FV (P,h)	92	92	0.03
Reformulated FOM	150	150	0.0137
ROM (4)	66	106	0.005
ROM (9)	81	127	0.0064

Identified the trade-off between the computational efficiency and accuracy of ROMs

Conclusions

- A reformulated Finite Volume heat exchanger model was developed.
- A nonlinear model order reduction framework was developed for heat exchanger applications.
- Reduced order model for a centrifugal chiller was generated and validated. The proposed POD-DEIM model reduction scheme led to 80% computation time reduction compared to a standard FV modeling approach.

Questions?

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